Isogeometric Analysis in electromagnetism: discretization with NURBS

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1 Introduction

Isogeometric Analysis (IGA) is a discretization technique introduced in [1] with the aim of improving the communications between CAD software and PDE solvers. The main idea in IGA is that the space of functions that describes the geometry in CAD, is also taken as the space of trial and test functions in the discrete solver. With this choice, and invoking the isoparametric concept, the geometry description given by CAD is exactly preserved at the coarsest level mesh.

The most widely diffussed functions in CAD, and the ones used in [1], are Non-Uniform Rational B-splines (NURBS). These are defined as quotient of B-splines, that is, of piecewise polynomials. Among other properties, NURBS basis functions are capable to describe conic sections. Compared to finite elements, the solution provided by NURBS is smoother, and the convergence in terms of the degrees of freedom is improved.

In [2] we introduced a set of discrete spaces satisfying a De Rham diagram in the context of IGA. These are B-spline spaces defined in the unit square (or cube), and then mapped to the physical domain through a NURBS parametrization, so that the CAD geometry is exactly preserved. The discretization properties of these spaces were studied in [3], by introducing suitable projectors that render the diagram commutative.

The definition of the previous spline spaces can be generalized to NURBS, but the commutativity properties with the differential operators are lost. Thus, the NURBS spaces do not satisfy a commuting diagram. However, they are well suited to discretize some differential problems, such as the mixed problems in electromagnetism. This can be proved using perturbation arguments. Indeed, the NURBS spaces do not satisfy a De Rham diagram but they are proved to be small perturbations of a complex.

In this work we will present a summary of our advances concerning NURBS discretizations, to appear in [4]. The talk will address both the theoretical results and the numerical tests. In particular, we will concentrate on Maxwell eigenvalue problems, and show that not all standard formulations stay spurious free when considering NURBS.

References

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